

# Phase velocity method for guided wave measurements in composite plates.

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2. Theoretical model
3. Materials and Methods
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# 1.- Introduction



## Objectives

- *Evaluation of composite plates using the phase velocity method applied to guided waves. (SV and SH).*
  - *Determination of elastic constant*
  - *Flaw evaluation (lamination)*

# 2.- Theoretical model

## 2.-Theoretical model

# Preliminary considerations

## **SV Waves (Lamb Waves)**

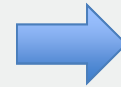
- 2D dimension (plane  $x_1, x_2$ )
- Orthotropic material (plane  $x_1, x_2$ )

## **SH Waves**

- Quasi-isotropic (plane  $x_1, x_3$ )

## 2.-Theoretical model

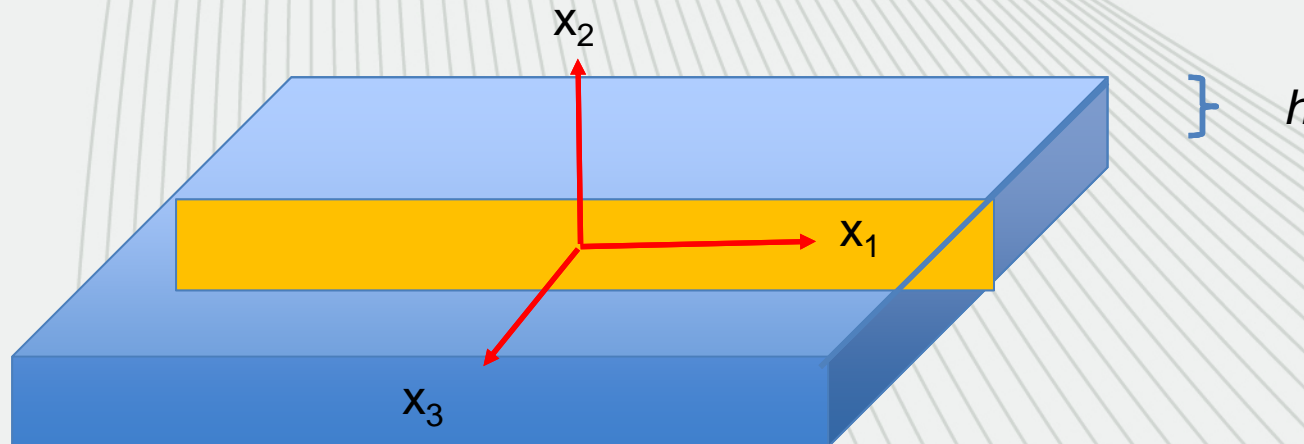
Anisotropic material



Voigt notation

$C_{11}$	$C_{12}$	$C_{13}$	0	0	0
	$C_{22}$	$C_{23}$	0	0	0
		$C_{33}$	0	0	0
			$C_{44}$	0	0
				$C_{55}$	0
					$C_{66}$

Stiffness Matrix





## 2.-Theoretical model

## SV Considerations

Voigt notation

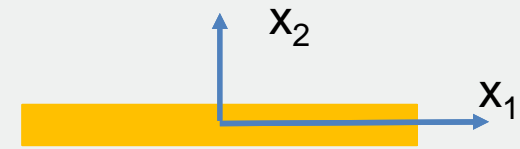
- $C_{11}$
- $C_{22}$
- $C_{12}$
- $C_{66}$

Engineering notation

$E_1$  and  $E_2$  Young modulus

$\mu_{12}$  and  $\mu_{21}$  Poisson ratio

$G_{12}$  Shear Modulus

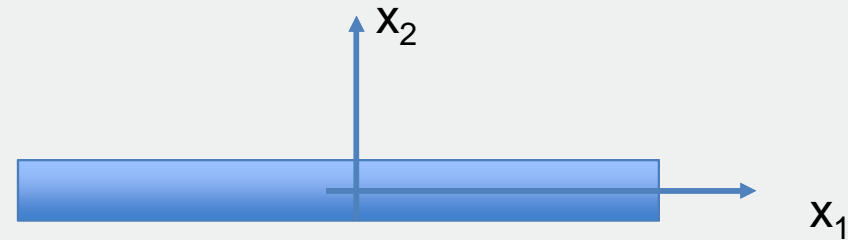


$$\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1}$$



## 2.-Theoretical model

Relation Voigt vs engineering notation  
(2D)



$$C_{11} = \frac{E_1}{(1 - \mu_{12}\mu_{21})} \quad C_{12} = \frac{\mu_{12}E_1}{(1 - \mu_{12}\mu_{21})} = \frac{\mu_{21}E_2}{(1 - \mu_{12}\mu_{21})}$$

$$C_{22} = \frac{E_2}{(1 - \mu_{12}\mu_{21})} \quad C_{66} = G_{12}$$

$$\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1}$$

## 2.-Theoretical model

Plane  $x_1 x_3$

Quasi-isotropic



$$E_1 = E_3$$
$$\mu_{13} = \mu_{31}$$

$$C_{33} = C_{44} = G_{13} = \frac{E_1}{(1 + \mu_{13})}$$

## 2.-Theoretical model

Waves equations in  $x_1$   $x_2$  plane

$$\rho \frac{\partial^2 u_1}{\partial t^2} = C_{11} \frac{\partial^2 u_1}{\partial x_1^2} + C_{12} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + C_{66} \frac{\partial^2 u_1}{\partial x_2^2} + C_{66} \frac{\partial^2 u_2}{\partial x_1 \partial x_2}$$
$$\rho \frac{\partial^2 u_2}{\partial t^2} = C_{12} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{22} \frac{\partial^2 u_2}{\partial x_2^2} + C_{66} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66} \frac{\partial^2 u_2}{\partial x_1^2}$$

$\rho$  = density

$u_1$  and  $u_2$  displacements



## 2.-Theoretical model

## Dispersion relations of SV (Lamb Waves)

symmetric

$$\frac{th \frac{sh}{2}}{th \frac{qh}{2}} = \frac{B_0 C_0}{A_0 D_0}$$

antisymmetric

$$\frac{th \frac{qh}{2}}{th \frac{sh}{2}} = \frac{B_0 C_0}{A_0 D_0}$$

$$\frac{qh}{2} = \pi \frac{h}{\lambda} \sqrt{-\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 - b}}$$

$$\frac{sh}{2} = \pi \frac{h}{\lambda} \sqrt{-\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - b}}$$

$$a = \frac{C_{12}^2}{C_{22}C_{66}} + \frac{2C_{12}}{C_{22}} - \frac{C_{11}}{C_{66}} + \rho c^2 \left( \frac{1}{C_{66}} + \frac{1}{C_{22}} \right)$$

$$b = \frac{C_{11}}{C_{22}} - \rho c^2 \frac{(C_{11} + C_{66})}{C_{22}C_{66}} + \rho^2 c^4 \left( \frac{1}{C_{66}C_{22}} \right)$$

$c =$  phase velocity

$\lambda =$  wavelength

## 2.-Theoretical model

Dispersion relations of SV (Lamb Waves)..  
cont..

$$A_0 = \frac{q}{k} \left[ -\rho c^2 + C_{11} - \frac{C_{12}(C_{12} + C_{66})}{C_{22}} \right] - C_{66} \left( \frac{q}{k} \right)^3$$

$$B_0 = \frac{s}{k} \left[ -\rho c^2 + C_{11} - \frac{C_{12}(C_{12} + C_{66})}{C_{22}} \right] - C_{66} \left( \frac{s}{k} \right)^3$$

$$C_0 = -\rho c^2 + C_{11} + C_{12} \left( \frac{q}{k} \right)^2$$

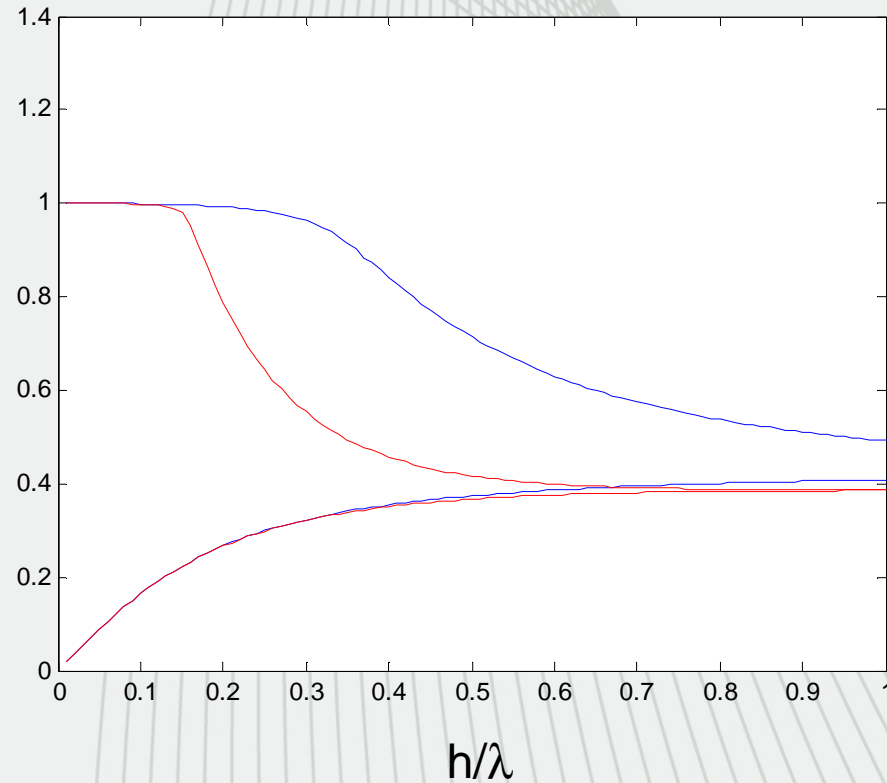
$$D_0 = -\rho c^2 + C_{11} + C_{21} \left( \frac{s}{k} \right)^2$$

$$k = 2\pi/\lambda$$

## 2.-Theoretical model

## Dispersion curves Case 1

$$C/C_1 \leftarrow C_1 = \sqrt{E_1/\rho}$$



Blue {  $\epsilon=0,5$   
 $\gamma=0,2$

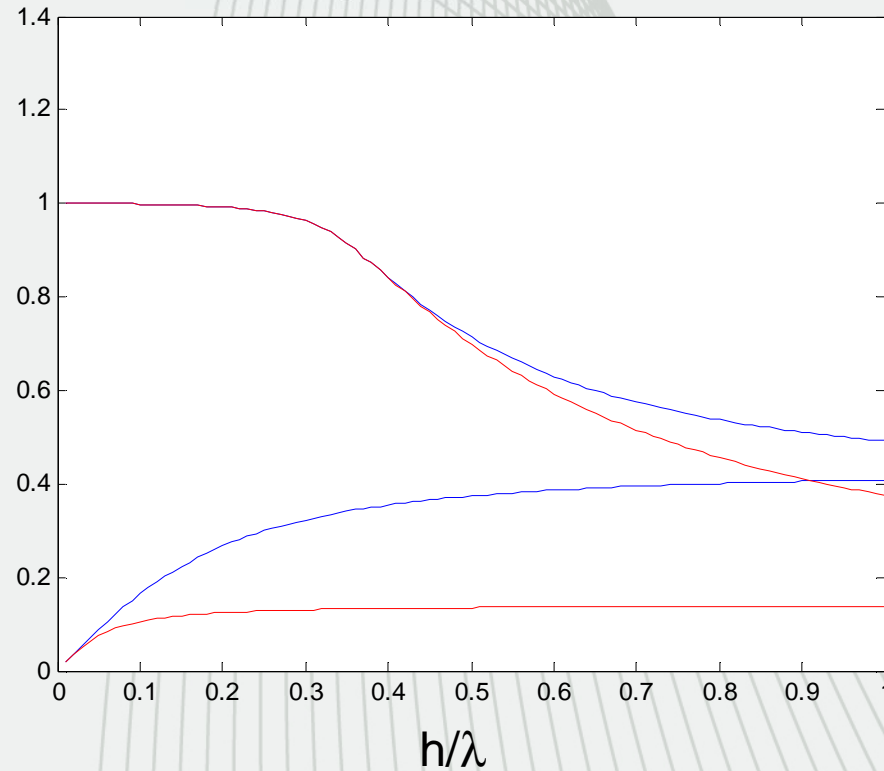
Red {  $\epsilon=0,1$   
 $\gamma=0,2$

$$\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1}$$



## 2.-Theoretical model

$$C/C_1 \leftarrow C_1 = \sqrt{E_1/\rho}$$



## Dispersion curves Case 2

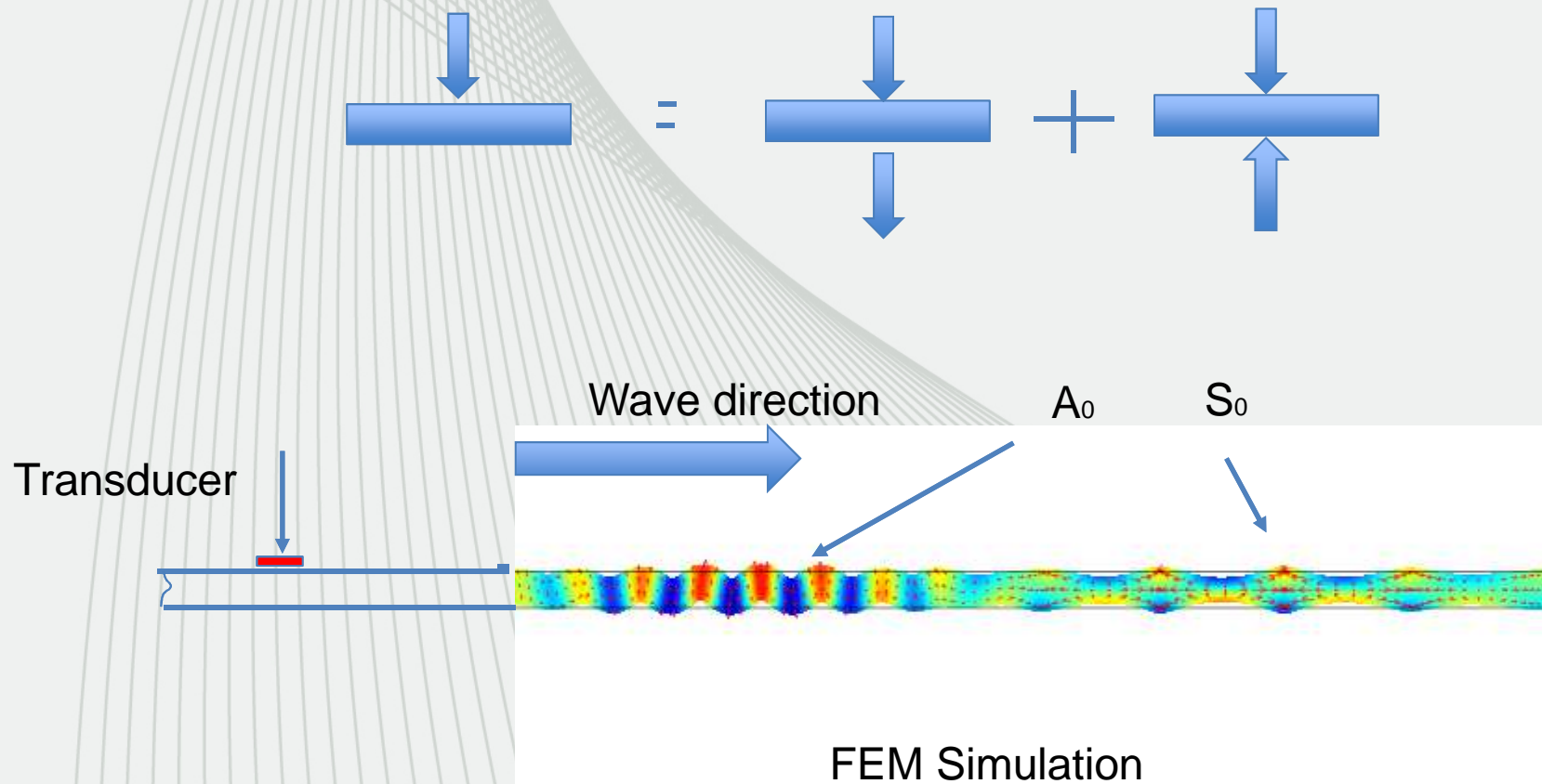
Blue {  $\epsilon=0,5$   
 $\gamma=0,2$

Red {  $\epsilon=0,5$   
 $\gamma=0,02$

$$\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1}$$

## 2.-Theoretical model

## Influence of Transducer (“one side”)



- Nieuwenhuis et al. Generation and detection of guided waves using PZT wafer transducer. <http://users.ece.cmu.edu/~dwg/research/Waves25rev.pdf>
- V. Giurgiutiu, “Lamb Wave Generation with Piezoelectric Wafer Active Sensors for Structural Health Monitoring,” Smart Structures and Materials 2003: Smart Structures and Integrated Systems, 111

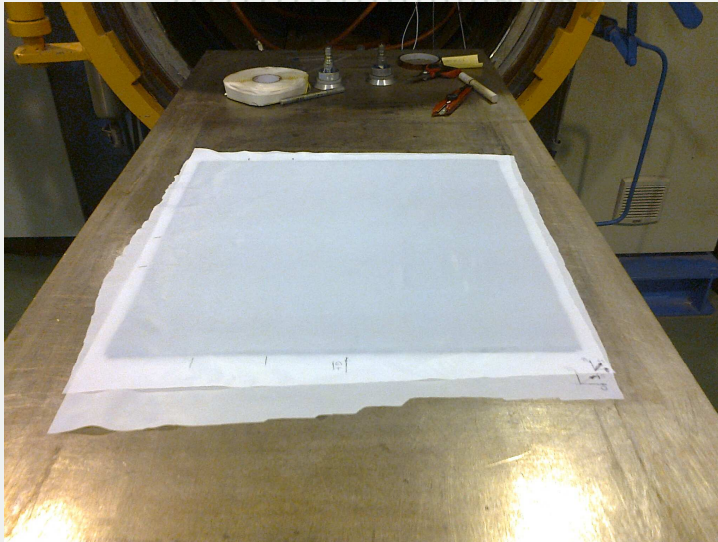
# 3.- Materials and Methods



### 3.-Materials and Methods

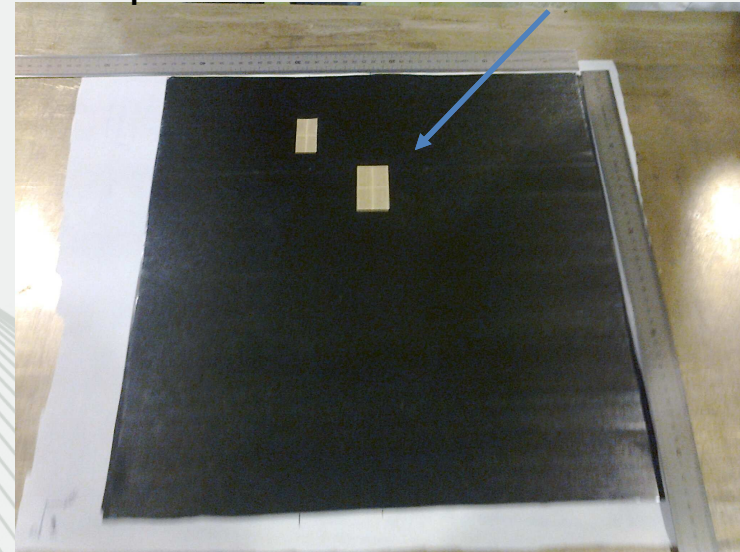
#### CFRP Laminates

Sample 1



660x460x2,9 mm

Sample 2



Cardboard

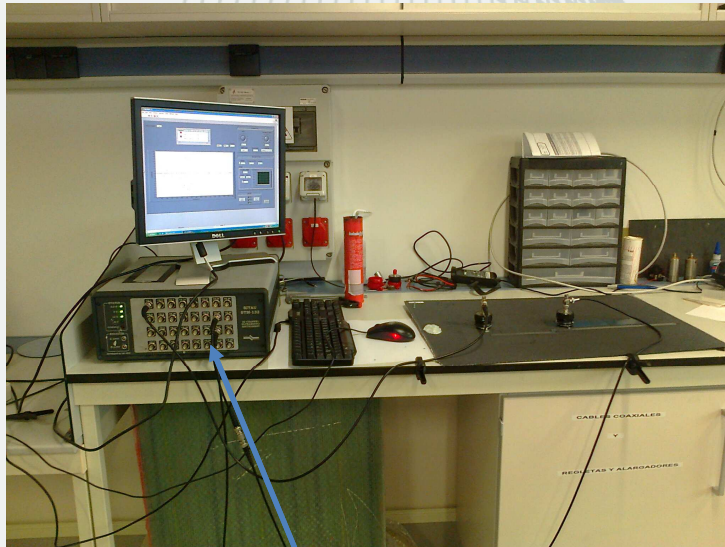
CFRP: -45/+45/90/0/0/90/+45/-45 / “ Symmetrical”  Quasi-symmetrical

$\rho=1,25 \text{ g/cm}^3$

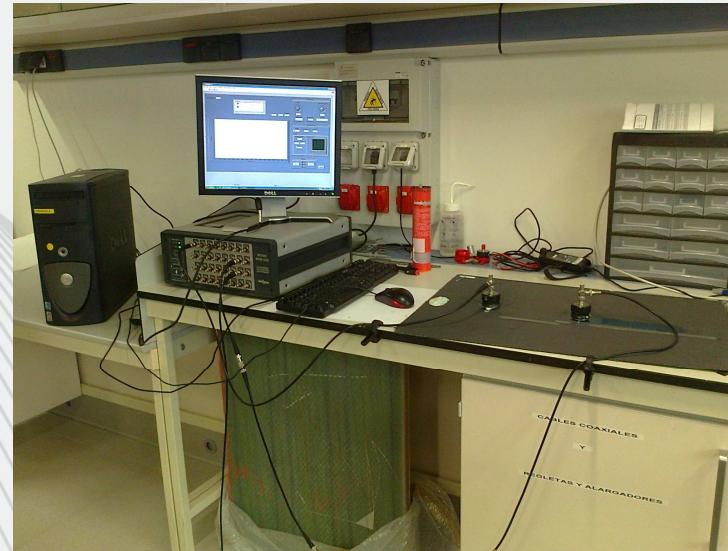


## 3.-Materials and Methods

### Equipment



Ultrasonic System Sitau , Dasel SL  
Labview GUI

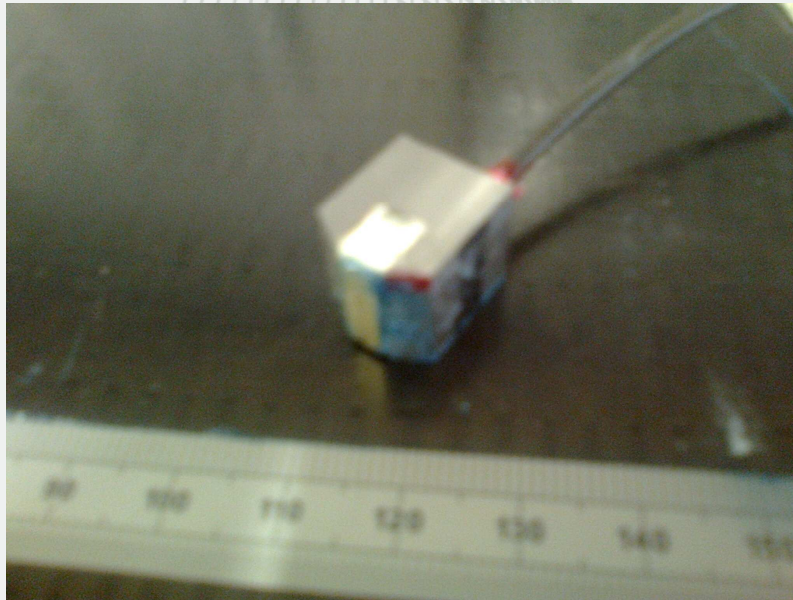


Burst excitation  
Pulse Transmission mode

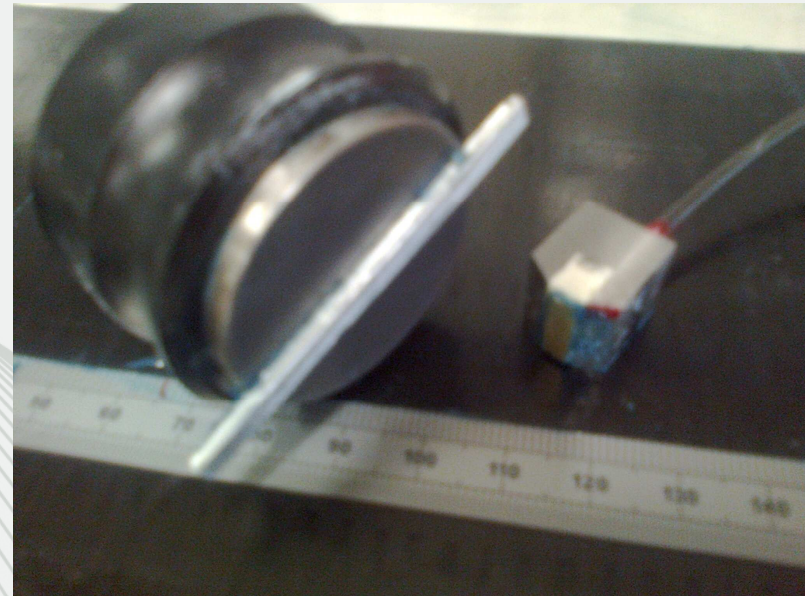


### 3.-Materials and Methods

#### Transducers



**SH Transducer  
(made in Tecnia)**

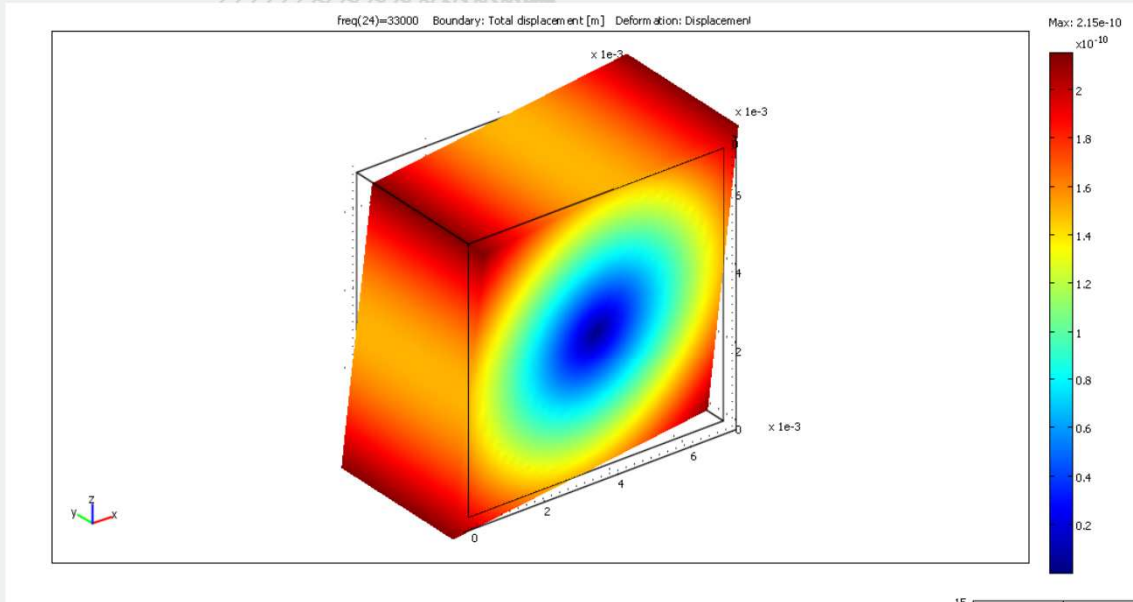


**SV – L Transducer  
Panametrics  
Rx= 0.5 MHz (V191)  
Tx= 1 MHz (V194)**



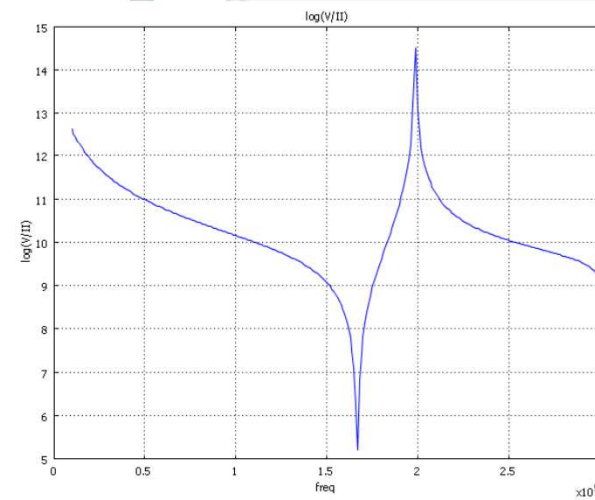
### 3.-Materials and Methods

### SH transducer. Piezoelectric material



**FEM Simulation**

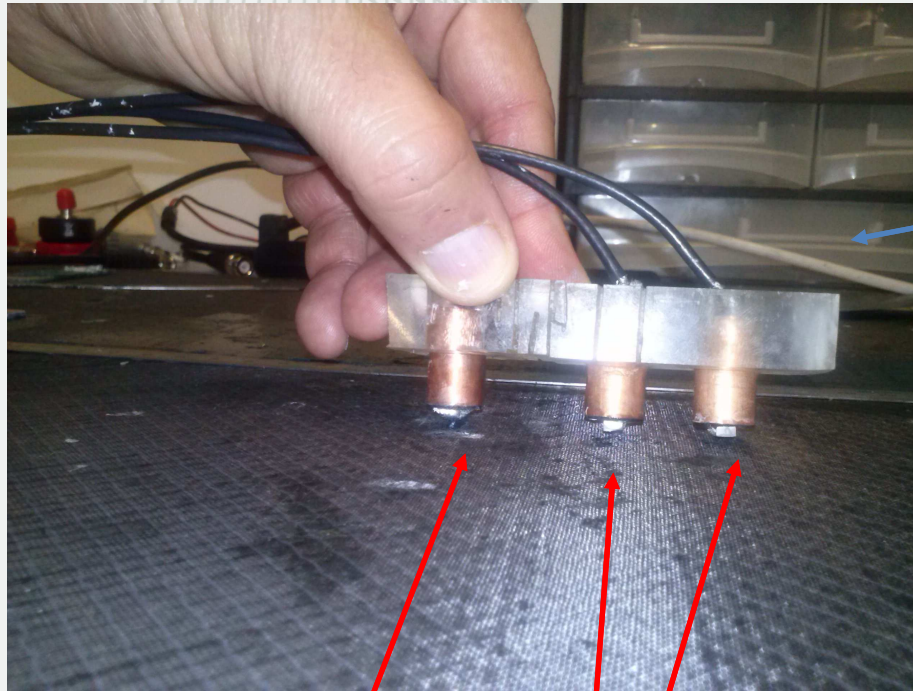
**Impedance module**



### 3.-Materials and Methods

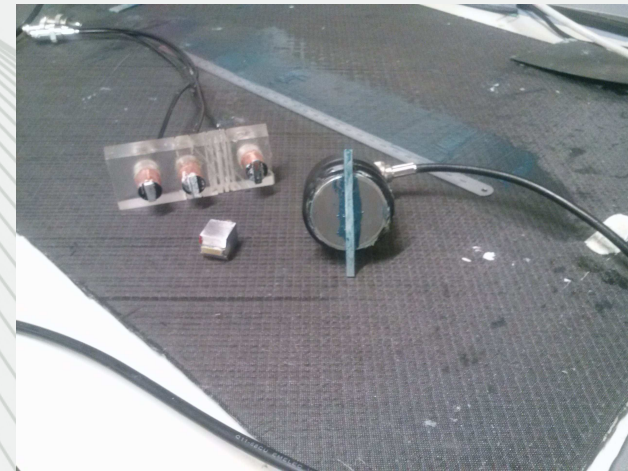
### Transducers

Transducers for  
flaw detection  
(longitudinal)



Tx

Rx



### 3.-Materials and Methods

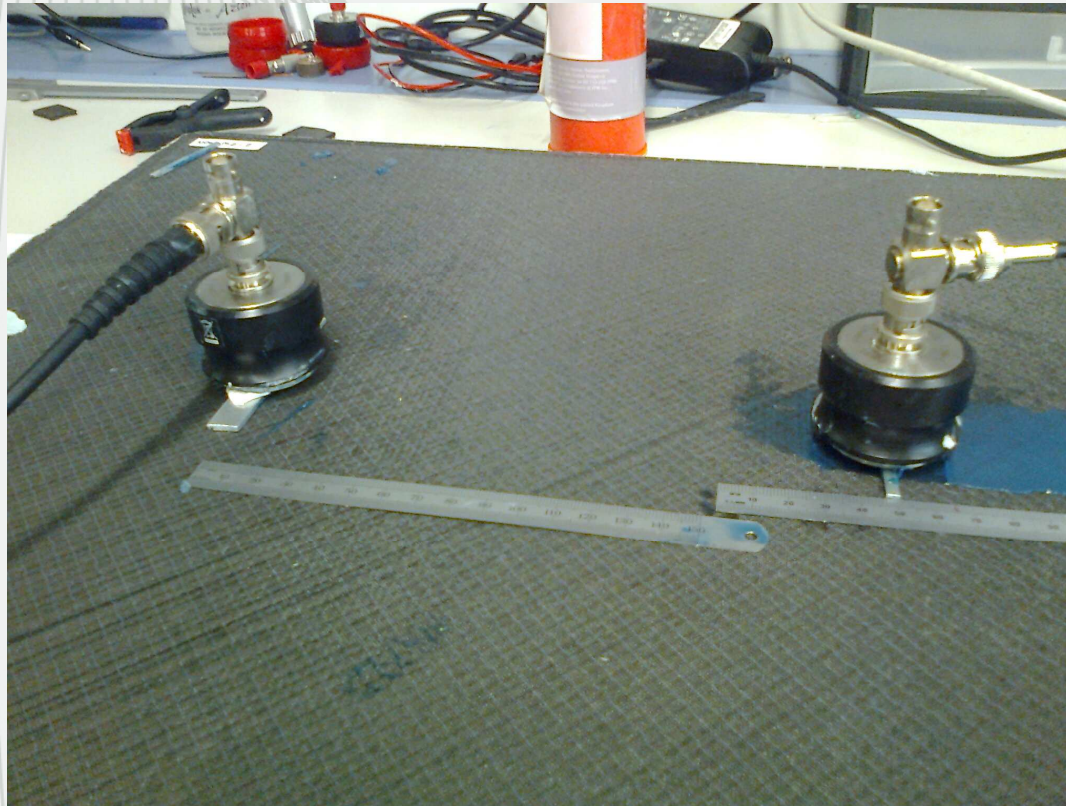
#### Basic Configuration





## 3.-Materials and Methods

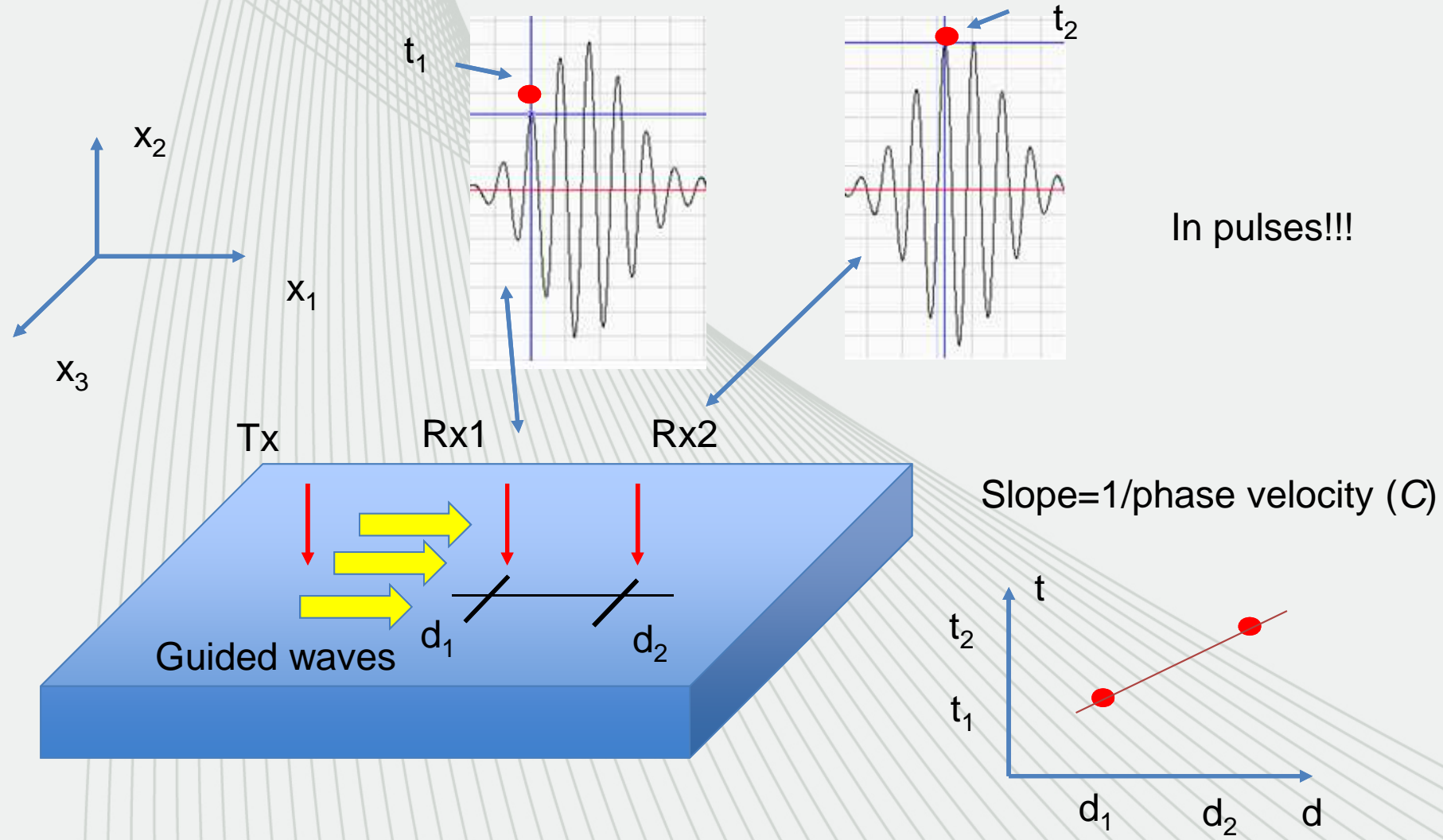
### Basic Configuration





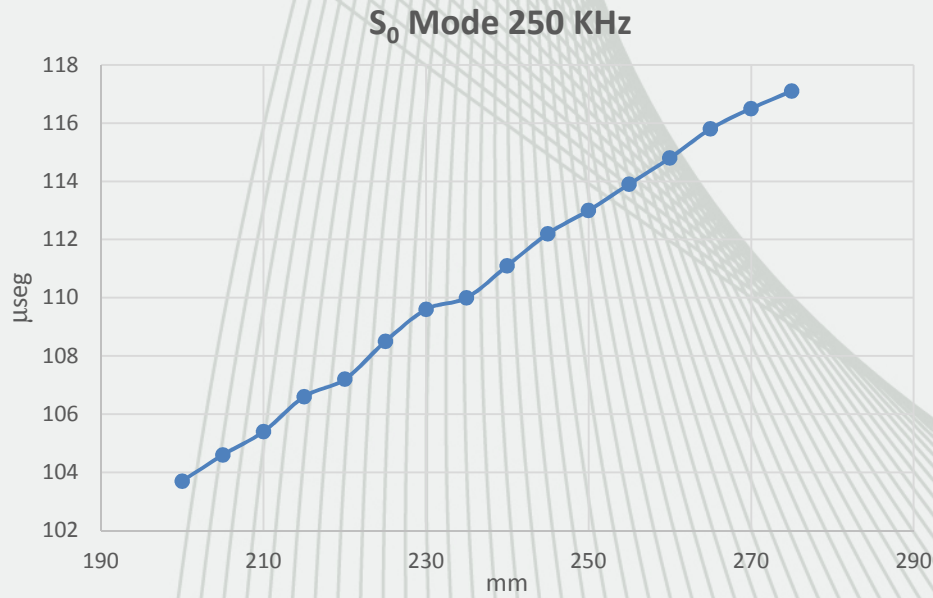
### 3.-Materials and Methods

### Phase velocity method

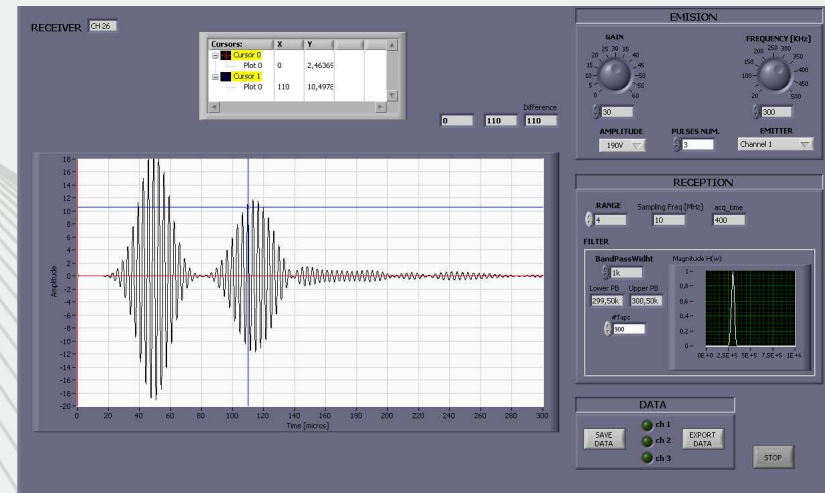


# 4.- Results

# 4.-Results



## S0 Mode



Sample No. 1

$$C_S = 5455 \pm 61 \text{ m/s}$$



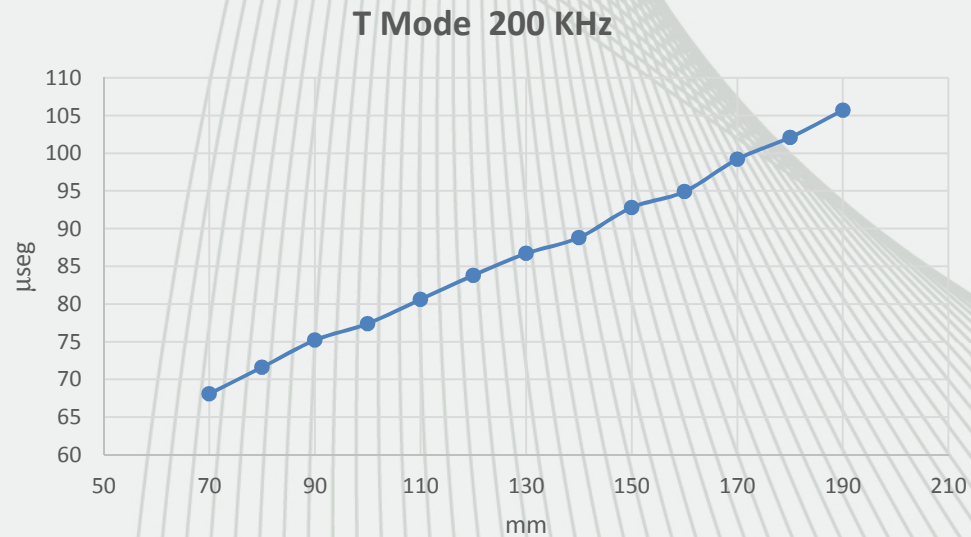
$C_1$



$$(h/\lambda) = 0.13 \text{ !!!!}$$

## 4.-Results

## Fundamental Poisson ratio



$$\mu_{13} = \frac{1}{2} \left( \frac{C_1}{C_T} \right)^2 - 1$$

$$\mu_{13} = 0,39$$

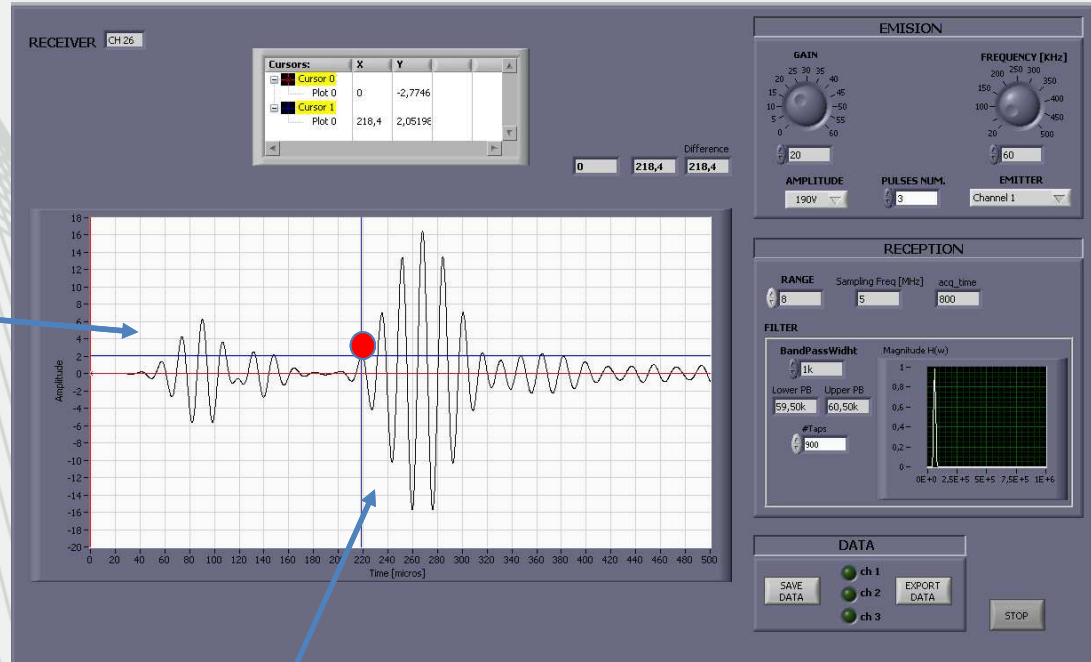
Sample No. 1

$$C_T = 3267 \pm 41 \text{ m/s}$$



# 4.-Results

## A<sub>0</sub> Mode. Evolution with distance



Initial Pulse

Tx

Rx

60 KHz

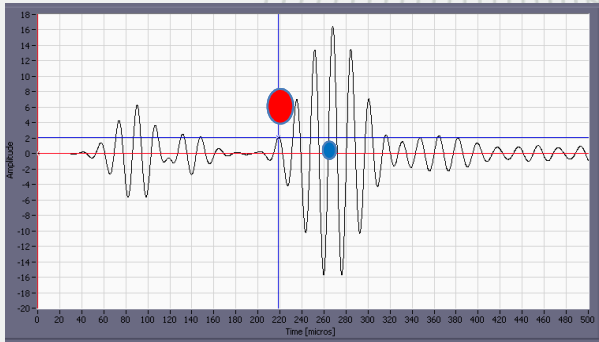
Sample No. 1

# 4.-Results

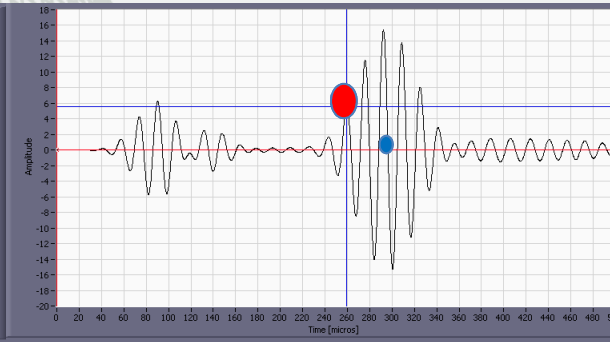
## Evolution with distance

- for phase velocity
- for group velocity

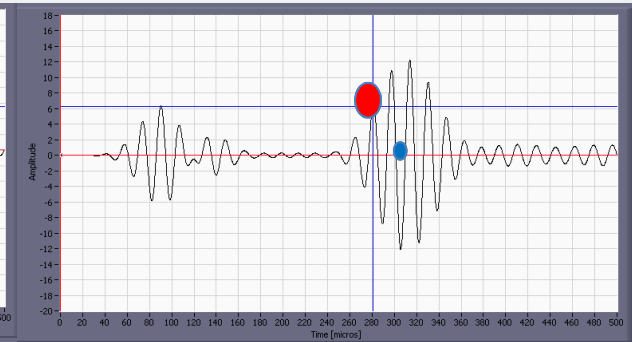
100 mm



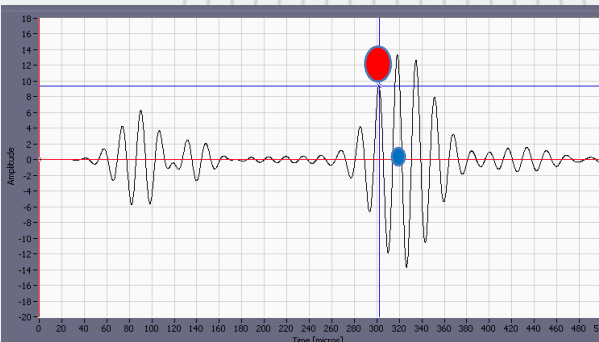
110 mm



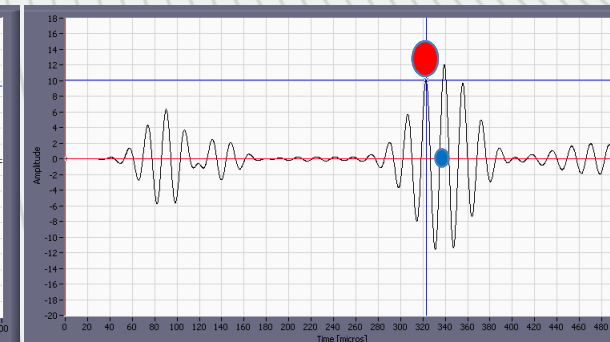
120 mm



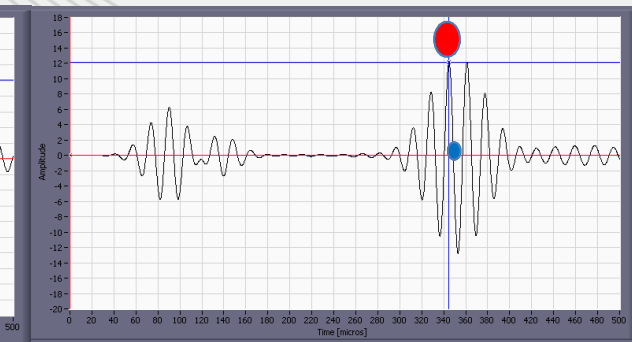
140 mm



160 mm



180 mm

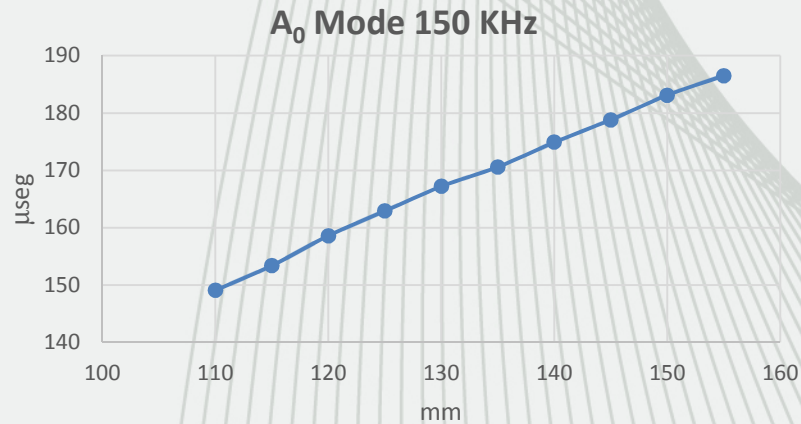


A<sub>0</sub> mode 60 KHz

Sample No. 1

## 4.-Results

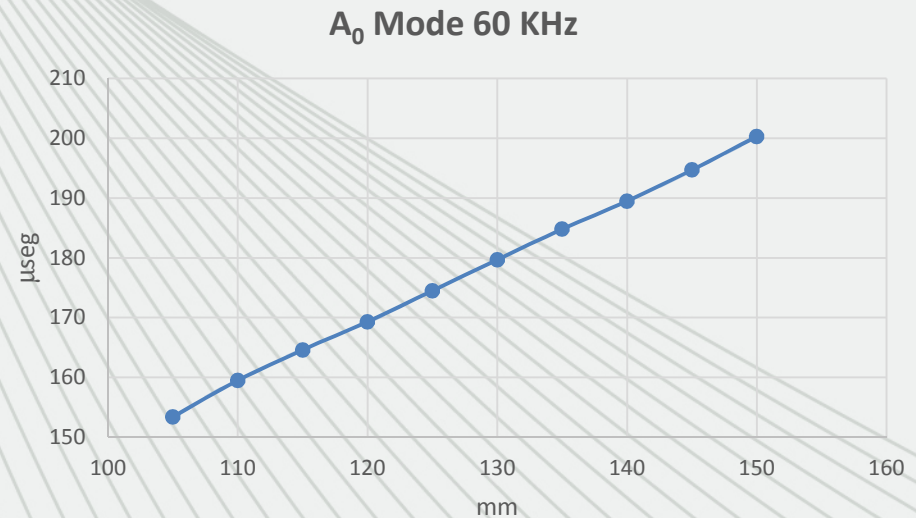
### A0 mode. Some examples



Sample No. 1

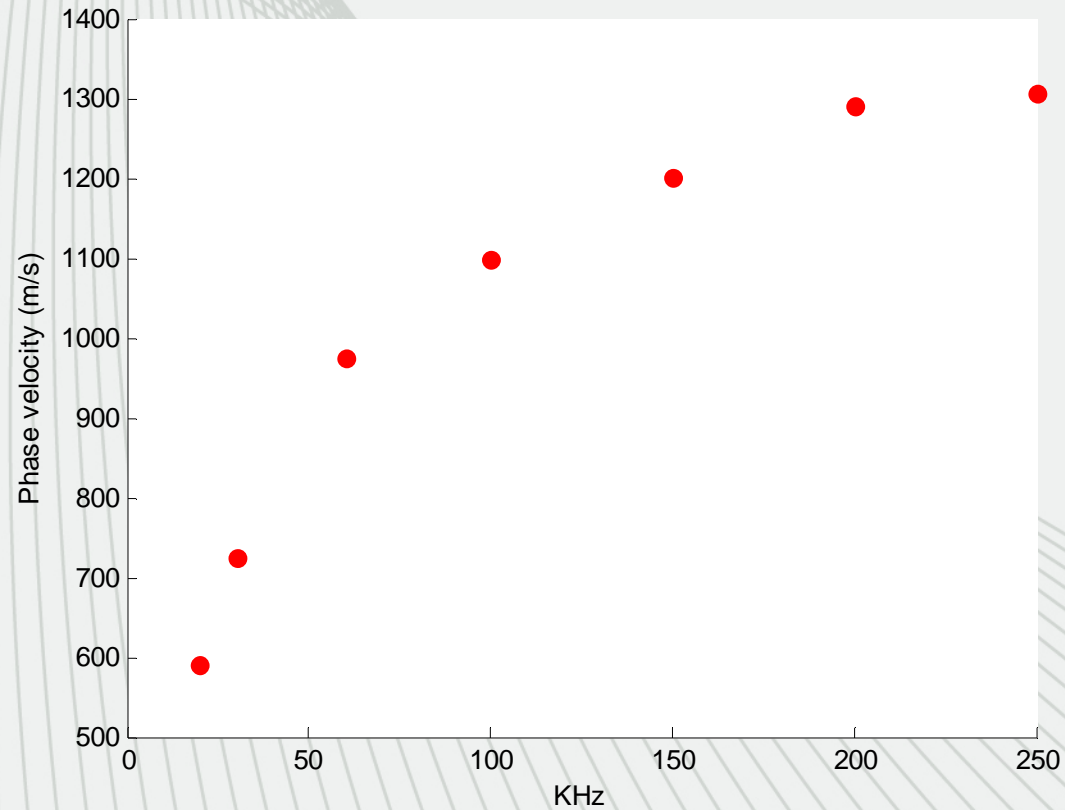
C=1202+/-18 m/s

976+/- 7 m/s



## 4.-Results

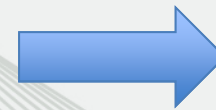
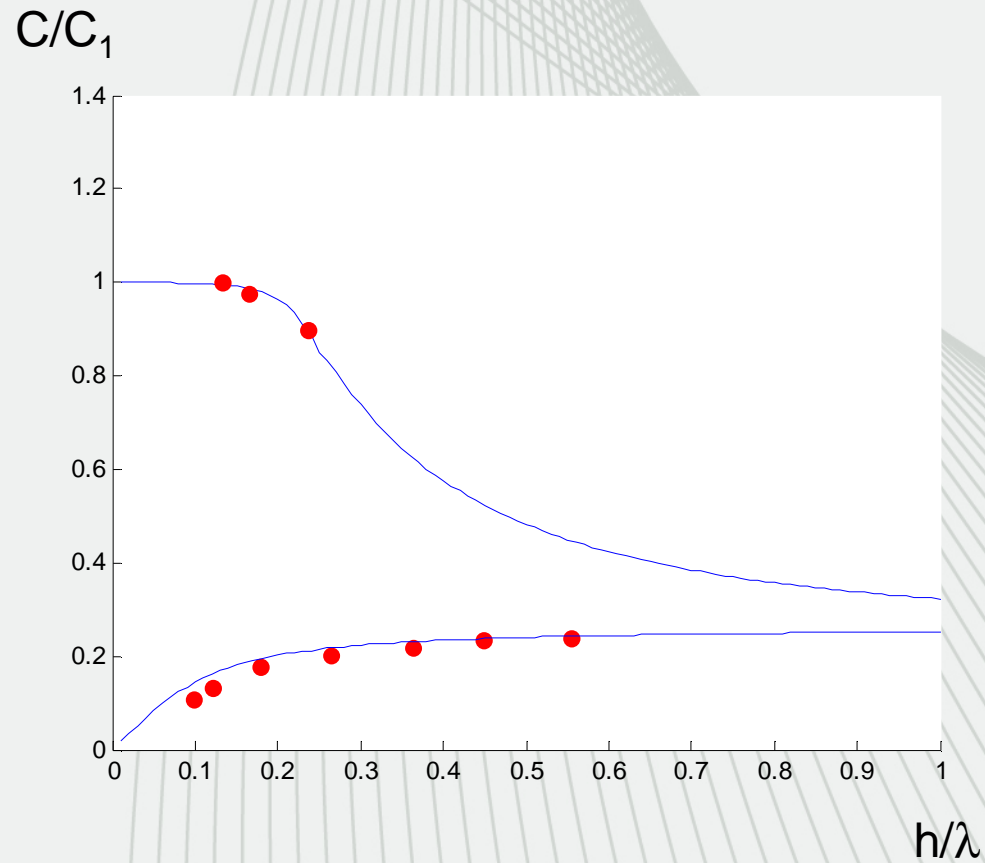
### $A_0$ Mode. Experimental dispersion curve Sample No 1





## 4.-Results

### Experimental vs theoretical



$$\begin{aligned}\varepsilon &= 0.2 \\ \gamma &= 0.07 \\ \mu_{12} &= 0.4\end{aligned}$$

## 4.-Results

### Elastic Constants obtained

#### Engineering constants

$E_1$ (GPa)	$E_2$ (GPa)	$G_{13}$ (Gpa)	$G_{12}$ (Gpa)	$\nu_{13}$
37,2	7,4	13,3	2,6	0,39

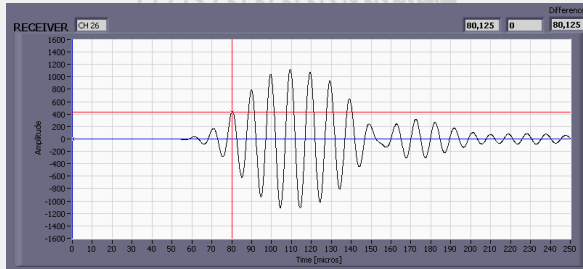
#### Voigt

$C_{11}$ (GPa)	$C_{22}$ (GPa)	$C_{12}$ (GPa)	$C_{66}$ (GPa)
38,4	7,7	3	2,6

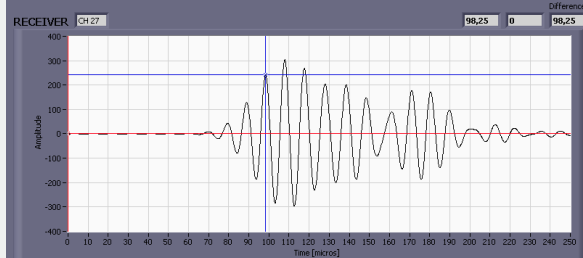
# 4.-Results

## Flaw evaluation

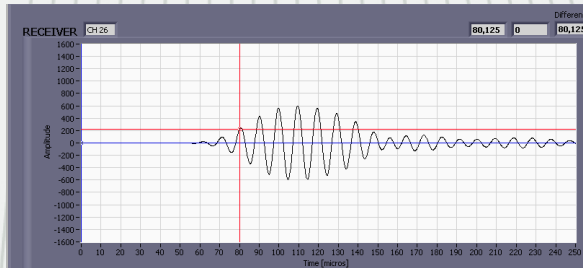
Rx1



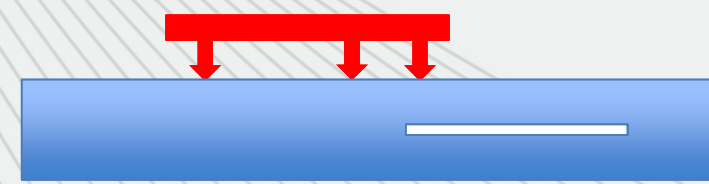
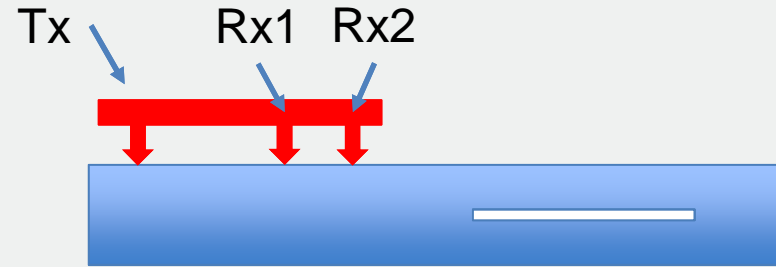
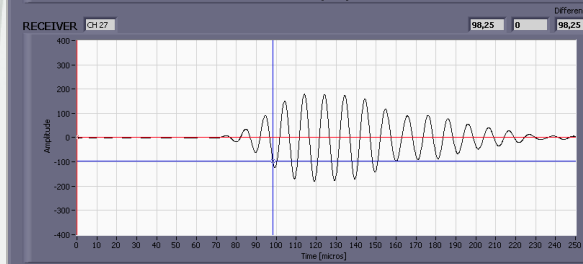
Rx2



Rx1



Rx2



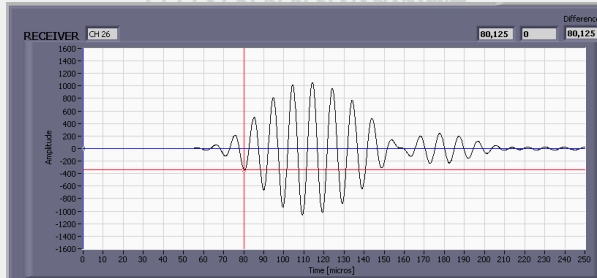
Sample No. 2



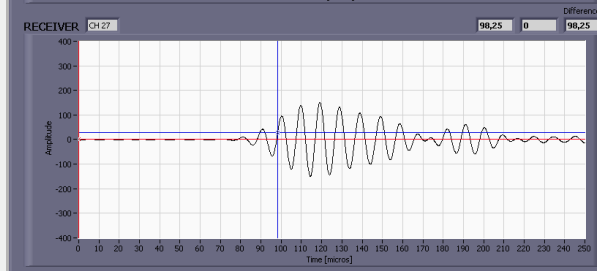
# 4.-Results

# Flaw evaluation

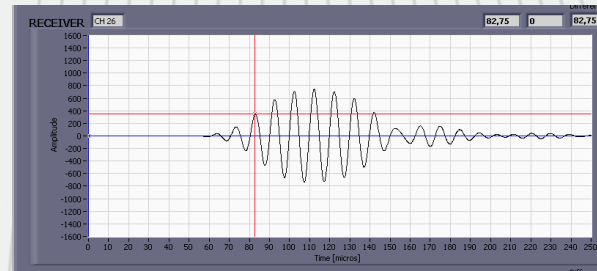
Rx1



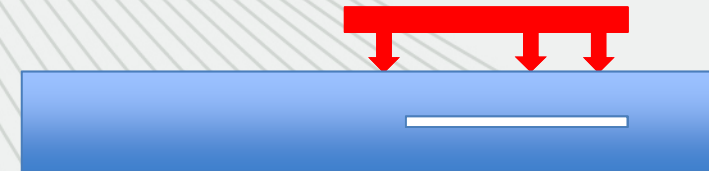
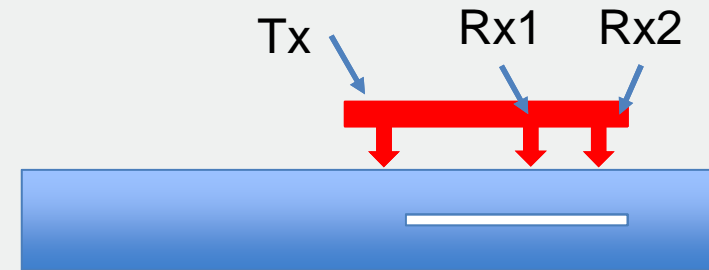
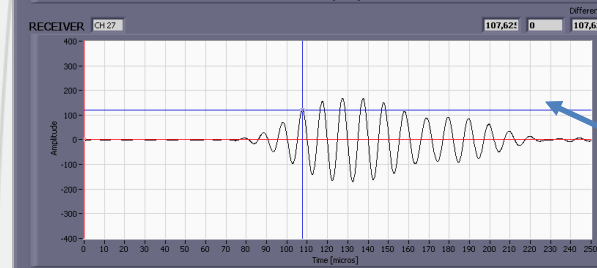
Rx2



Rx1



Rx2



New cursor position

Sample No. 2

## 4.-Results

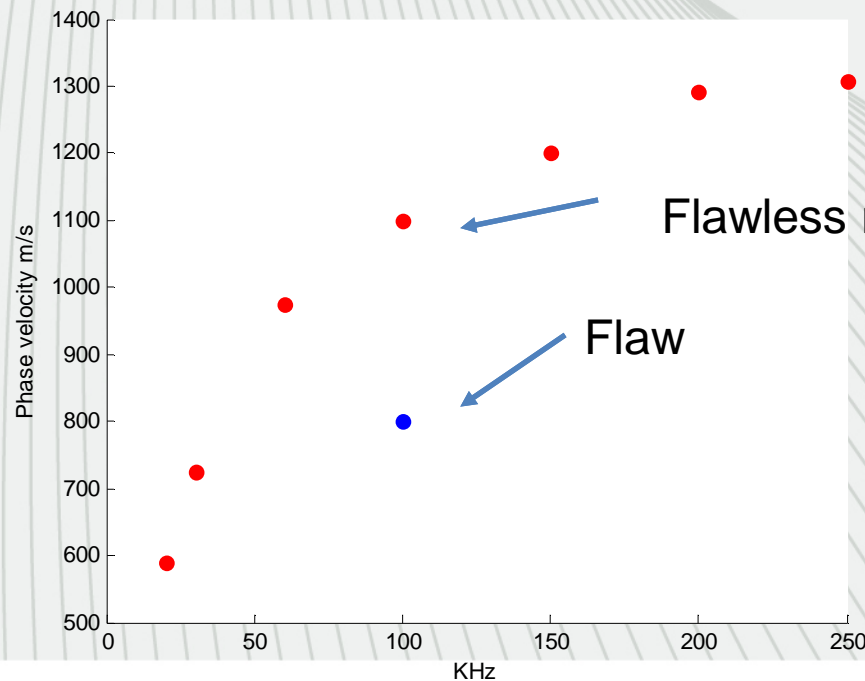
## Numerical evaluation

If  $c=1.1 \text{ mm}/\mu\text{seg}$  at 100 KHz

Then  $A=\text{Time difference in flawless position}=18,125 \mu\text{seg}$

$B=\text{Time difference in flaw position}=24.875 \mu\text{seg}$

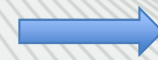
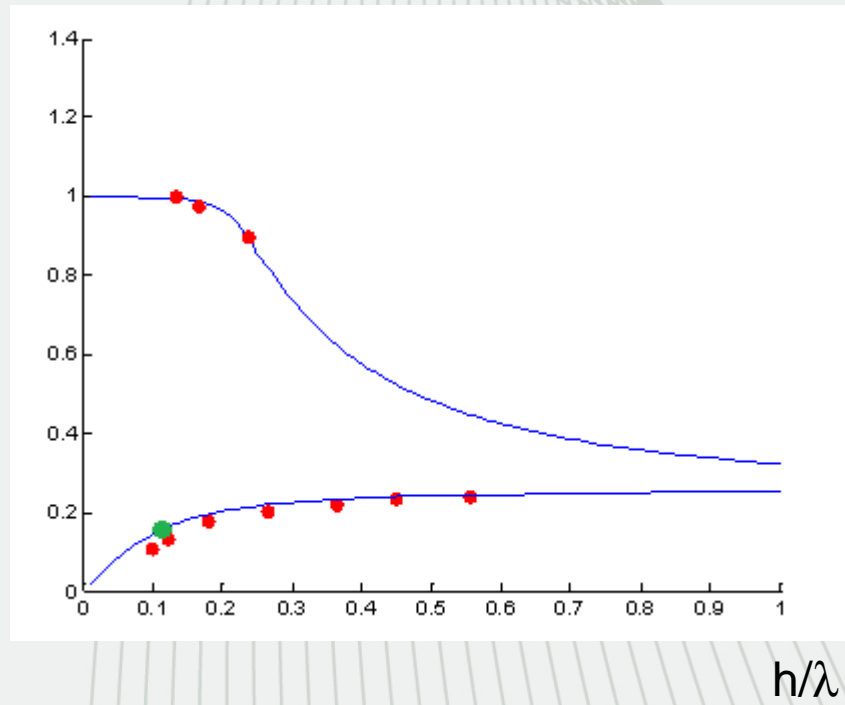
**Velocity over the flaw=  $(A/B)*1,1= 0,801 \text{ mm}/\mu\text{seg}= 801 \text{ m/s}$**



## 4.-Results

### Numerical evaluation vs. theoretical model

$C/C_1$



$h=1\text{mm}$  in the flaw region



# Conclusions

- The phase velocity method was used for determination of elastic constants and the evaluation of plates.
- It was possible to use this method with SH and SV guided waves.
- A flaw evaluation (lamination) could be possible with this method.
- But the elastic model should be changed. (viscoelastic model)

## Some extra details

- The “elastic” constants should be evaluated as viscoelastic “constants”

$$C_{ij} = C_{ijR} + iC_{ijI}$$



$$C_{ij}(\omega) = C_{ijR}(\omega) + iC_{ijI}(\omega)$$



$$C_{ijI}(\omega) \longleftrightarrow C_{ijR}(\omega)$$

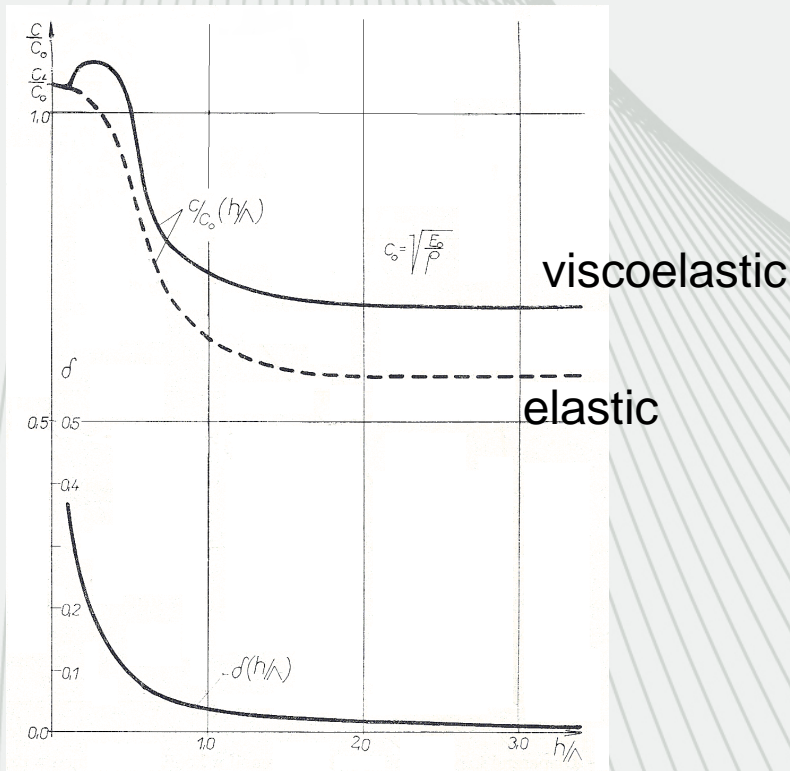
Dispersion

- Geometrical dispersion
- Viscoelastic dispersion

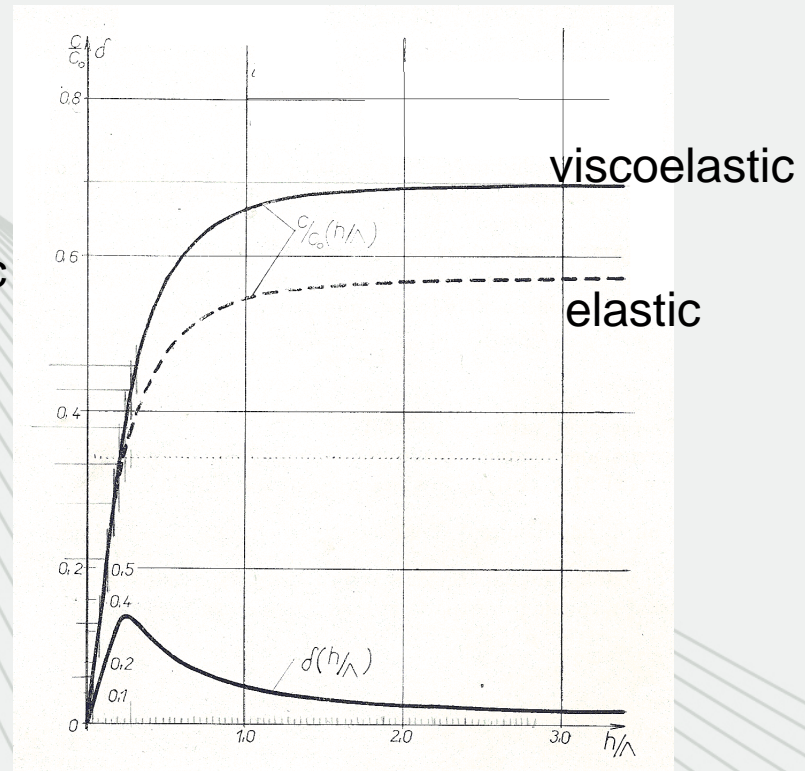
# Some extra details

# Viscoelastic dispersion model

From.:Martincek G. Theory and Methods of Dynamic Nondestructive Testing of Plane Elements. VEDA , Bratislava 1975



symmetrical



antisymmetrical



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# Thank you

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